

Date: April 21, 2006, Friday

Time: 10:40-11:30

Math 114 Calculus – Quiz I – Solutions

Q-1) Reverse the order of integration $\int_0^4 \int_{y/2}^{\sqrt{y}} dx dy$. Do not evaluate the integral.

Solution: $\int_0^4 \int_{y/2}^{\sqrt{y}} dx dy = \int_0^2 \int_{x^2}^{2x} dy dx$.

Q-2) Find the area of the region in the upper half plane bounded by the curves $y = -x^2 - 2x$, $y = 4 - x^2$ and $y = 3x$.

Solution:

$$\begin{aligned} \text{Area} &= \int_{-2}^0 \int_{-x^2-2x}^{4-x^2} dy dx + \int_0^1 \int_{3x}^{4-x^2} dy dx \\ &= \int_{-2}^0 (4 + 2x) dx + \int_0^1 (4 - x^2 - 3x) dx \\ &= \left(4x + x^2 \Big|_{-2}^0\right) + \left(4x - x^3/3 - 3x^2/2 \Big|_0^1\right) \\ &= 4 + \frac{13}{6} = \frac{37}{6}. \end{aligned}$$

Q-3) Write the integral $\int_{-2}^2 \int_{-\sqrt{2-x^2}/2}^{\sqrt{2-x^2}/2} \int_{x^2+3y^2}^{8-x^2-y^2} dz dy dx$ in the order $dx dz dy$. Do not evaluate.

Solution:

$$\int_{-2}^2 \int_{-\sqrt{2-x^2}/2}^{\sqrt{2-x^2}/2} \int_{x^2+3y^2}^{8-x^2-y^2} dz dy dx = \int_{-\sqrt{2}}^{\sqrt{2}} \int_{3y^2}^{4+y^2} \int_{-\sqrt{z-3y^2}}^{\sqrt{z-3y^2}} dx dz dy + \int_{-\sqrt{2}}^{\sqrt{2}} \int_{4+y^2}^{8-y^2} \int_{-\sqrt{8-y^2-z}}^{\sqrt{8-y^2-z}} dx dz dy.$$

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