Date: June 19, 2009, Friday NAME:.....

STUDENT NO:.....

SECTION NUMBER: .....

Math 116 Calculus – QUIZ # 2

Q-1) Write the equations of the tangent lines to the ellipse $3x^2 + 2xy + 2y^2 + x - 7y = 7$ at the two points on it corresponding to $x = -1$.

Solutions: Let $f(x, y) = 3x^2 + 2xy + 2y^2 + x - 7y - 7$. The equation of the tangent line at the point (x_0, y_0) on the ellipse $f(x, y) = 0$ is

$$\nabla f(x_0, y_0) \cdot (x - x_0, y - y_0) = 0.$$

When $x = -1$, we find the y-coordinate from the equation $f(-1, y) = 2y^2 - 9y - 5 = 0$ as $y = 5$ and $y = -1/2$.

We calculate $\nabla f = (6x + 2y + 1, 2x + 4y - 7)$, so

$$\nabla f(-1, 5) = (5, 11) \text{ and } \nabla f(-1, -1/2) = (-6, -11).$$

The required equations for the tangent lines are then

$$(5, 11) \cdot (x + 1, y - 5) = 0, \text{ or } 5x + 11y = 50,$$

and

$$(-6, -11) \cdot (x + 1, y + \frac{1}{2}) = 0, \text{ or } 6x + 11y = -\frac{23}{2}.$$

When $x = 1$, then the corresponding points are $(1, 3)$ and $(1, -1/2)$. We then have

$$\nabla f(1, 3) = (13, 7), \text{ and } \nabla f(1, -1/2) = (6, -7).$$

The equations of the tangent lines are

$$(13, 7) \cdot (x - 1, y - 3) = 0, \text{ or } 13x + 7y = 34,$$

and

$$(6, -7) \cdot (x - 1, y + 1/2) = 0, \text{ or } 6x - 7y = -19/2.$$