Date: July 10, 2009, Friday

NAME:..

STUDENT NO:..

SECTION NUMBER: ....

Math 116 Calculus – QUIZ # 8 – Solutions

Question: Show that the vector field $\mathbf{F} = xy^2\,\mathbf{i} + x^2y\,\mathbf{j}$ is conservative and find its potential function.

Solution: Let $\mathbf{F} = (M, N)$. $\mathbf{F}$ is conservative since $M_y = N_x$. Let $f(x, y)$ be a potential function for $\mathbf{F}$. We find f as follows.

$$\begin{aligned}
f_x(x,y) &= M = xy^2. \\
f(x,y) &= \frac{1}{2}x^2y^2 + \phi(y). \\
f_y(x,y) &= x^2y + \phi'(y) = N = x^2y, \text{ so } \phi(y) = c \text{ is constant.} \\
f(x,y) &= \frac{1}{2}x^2y^2 + c, \text{ where } c \text{ is any constant.}
\end{aligned}$$

Question: Show that the vector field $\mathbf{F} = xy^2\,\mathbf{i} + x^2y\,\mathbf{j} + 2z\mathbf{k}$ is conservative and find its potential function.

Let $\mathbf{F} = (M, N, P)$. $\mathbf{F}$ is conservative since $M_y = N_x$, $M_z = P_x$ and $N_z = P_y$. Let $f(x, y, z)$ be a potential function for $\mathbf{F}$. We find f as follows.

$$\begin{aligned}
f_x(x,y,z) &= M = xy^2. \\
f(x,y,z) &= \frac{1}{2}x^2y^2 + \phi(y,z). \\
f_y(x,y,z) &= x^2y + \phi_y(y,z) = N = x^2y, \text{ so } \phi(y,z) = \alpha(z) \text{ is constant with respect to } y. \\
f(x,y,z) &= \frac{1}{2}x^2y^2 + \alpha(z). \\
f_z(x,y,z) &= \alpha'(z) = P = 2z, \text{ so } \alpha(z) = z^2 + c, \text{ and} \\
f(x,y,z) &= \frac{1}{2}x^2y^2 + z^2 + c, \text{ where } c \text{ is any constant.}
\end{aligned}$$