Math206 HW#8 Solutions

1)

```
>> syms t
>> laplace(t^3+2*t^2+-t+1)

ans =

6/s^4+4/s^3-1/s^2+1/s

>> laplace((t^2+2*t-1)*exp(2*t))

ans =

2/(s-2)^3+2/(s-2)^2-1/(s-2)
```

2) a) Taking the Laplace transform of both sides, we obtain;

$$\frac{d^2}{dt^2} f(t) - \frac{d}{dt} f(t) - 2f(t) = \delta(t-1)$$

$$(s^2F(s) - sf(0) - f'(0)) - (sF(s) - f(0)) - 2F(s) = e^{-s}$$

$$(s^2 - s - 2)F(s) = e^{-s}$$

$$F(s) = \frac{e^{-s}}{(s^2 - s - 2)}$$

b) Using MATLAB to inverse transform the result,

```
>> syms s
>> ilaplace(exp(-s)/(s^2-s-2))

ans =

-1/3*Heaviside(t-1)*exp(-t+1)+1/3*Heaviside(t-1)*exp(2*t-2)

>> simple(ans)

...
...

ans =

1/3*Heaviside(t-1)*(-exp(-t+1)+exp(2*t-2))
```

We see that the result involves Heaviside functions, which is expected as the derivative of Heaviside functions yield the Delta-Dirac impulse function.

3) Since we are on the circle of radius ½, $|z| = \frac{1}{2}$ in the region. Choose:

$$f(z) = 7z^2$$
$$g(z) = 4z^5 - 1$$

Taking absolute values;

$$|f(z)| = |7z^2| = 7 \cdot \left(\frac{1}{2}\right)^2 = 1.75$$
$$|g(z)| = |4z^5 - 1| \leq 4 \cdot \left(\frac{1}{2}\right)^5 + 1 = 1.125$$

→ Thus $|f(z)| > |g(z)|$ on the given circle and since $f(z)$ has 2 roots inside, so does $f(z) + g(z)$. Therefore inside the circle of radius ½ there are two roots of the equation $4z^5 + 7z^2 - 1 = 0$.

Verification with MATLAB :

```
>> roots([4 0 0 7 0 -1])

ans =

   0.5863 + 1.0774i
   0.5863 - 1.0774i
  -1.1609
   0.3725
  -0.3842

>> abs(ans)

ans =

    1.2266
    1.2266
    1.1609
    0.3725
    0.3842
```

→ Thus only **2** roots exist inside the circle of radius ½ .

4) Using MATLAB to evaluate the integral,

```
>> syms x
>> int(sin(x)/x,0,inf)

ans =

1/2*pi
```

This is same as the result seen on page 220 of the book.