MATH 302 Complex Analysis II
Spring 2016

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This course will apply the basic knowledge of complex calculus to numerous interesting problems.

I will remind and review the necessary background every time we need to use them so only a nodding acquaintance with complex calculus is more than enough to follow the course.

Here is a sampling of the interesting problems that we will consider in the course.

- We know that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, but what is $\sum_{n=1}^{\infty} \frac{1}{n^{2k}}$ for $k = 3, 4, \ldots$.
- Just demand that a complex valued map from an open subset U of $\mathbb{C}$ preserve angles. Then be amazed that you are talking about invertible holomorphic maps. Restrict the subset further and you get only the Mobius transformations.
- Continuing from the previous item, how many different *nice* U do we have that are different? Riemann says there are only two.
- Is the maximum modulus of a holomorphic function always achieved on the boundary of the domain even when the domain is not bounded?
- Real and imaginary parts of a holomorphic function are harmonic. But harmonic functions have a life of their own. Can we recognize a harmonic function by just looking at its values on the boundary of its domain even when it is no more harmonic on the boundary?
- The crown achievements of Euler and Weierstrass: Can we recover an entire function by knowing only its zeros? In particular how many entire functions can we have which vanish exactly at the zeros of the sine function?
- The real Gamma function extends the factorial function from integers to reals. The complex Gamma function now extends the real Gamma function to the complexes and has marvelous properties.
- The most famous and the most expensive problem in mathematics, with a one million Dollar prize on it, is finding all the zeros of the Riemann zeta function $\zeta(z)$ which in turn satisfies a functional equation with the complex Gamma function and the sine function.
- The function $\pi(x)$ which gives the number of primes less than or equal to x has not been formulated properly yet but we nonetheless know that it is very close to $x/\ln(x)$ as x grows. This is the celebrated Prime Number Theorem.

Incidentally in the first item of the above list we are asking for the values of $\zeta(2k)$.

What about $\zeta(2k + 1)$?

For the syllabus and other technical details visit my web page.
http://sertoz.bilkent.edu.tr/

$$\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^z} = \prod_{p \text{ prime}} \frac{1}{1 - p^{-z}}, \quad \operatorname{Re} z > 1.$$