Due Date: 14 May 2015, Thursday
Time: Until 17:30
Instructor: Ali Sinan Sertöz

NAME:..

STUDENT NO:.......................................

Math 430 / Math 505 Introduction to Complex Geometry – Homework 5 – Solutions

1	2	3	4	5	TOTAL
50	50	0	0	0	100

Please do not write anything inside the above boxes!

Check that there are **2** questions on your exam booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit. **Submit your solutions on this booklet only. Use extra pages if necessary.**

Rules for Homework and Take-Home Exams

(1) You may discuss the problems with your classmates or with me but it is absolutely mandatory that you write your answers alone. Any similarity with your written words with any other solution or any other source that I happen to know is a direct violation of honesty.

(2) In particular do not lend your written solutions to your friends, nor borrow your friends's written solutions. Oral exchange of ideas is acceptable and is in fact encouraged.

(3) You must obey the usual rules of attribution: all sources you use must be explicitly cited in such a manner that the source can be easily retrieved by the reader. This includes any ideas you borrowed from your friends.

(4) Finally, in your written solution make sure that you exhibit your total understanding of the ideas involved, even mentioning where you quote a result but don't really follow the reasoning. This is an essential ingredient of learning.

Affidavit of compliance with the above rules: *I affirm that I have complied with the above rules in preparing this submitted work. Every solution I wrote reflects my true understanding of the problem. Any sources used, ideas from friends or others are explicitly cited without exception.*

Please sign here:

NAME: STUDENT NO:

Q-1) Let K be the canonical divisor of $\mathbb{P}^n$ and H a hyperplane divisor. Show that $K = (-n-1)H$.

Solution:

Let $[z_0 : \cdots : z_n]$ be the homogeneous coordinates on $\mathbb{P}^n$. We define three Euclidean charts.

$$x_1 = \frac{z_1}{z_0},\ x_2 = \frac{z_2}{z_0},\ \ldots, x_n = \frac{z_n}{z_0}$$
$$y_1 = \frac{z_0}{z_1},\ y_2 = \frac{z_2}{z_1},\ \ldots, y_n = \frac{z_n}{z_1}$$
$$w_1 = \frac{z_0}{z_2},\ w_2 = \frac{z_1}{z_2},\ w_3 = \frac{z_3}{z_2} \ldots, w_n = \frac{z_n}{z_2}$$

Check that

$$dy_1 \wedge \cdots \wedge dy_n = -\frac{1}{x_1^{n+1}} dx_1 \wedge \cdots dx_n = -\frac{1}{w_2^{n+1}} dw_1 \wedge \cdots dw_n.$$

This can be continued to the other charts to conclude that the divisor of this canonical form is $(-n-1)H$ where H is the hyperplane given by the vanishing of x_1. Since all canonical divisors are linearly equivalent, we can write

$$K = (-n-1)H.$$

NAME: STUDENT NO:

Q-2) Let M be a compact complex Kahler manifold and let $L_1,\ L_2$ be two line bundles on M with L_1 being positive. Show that for all sufficiently large integers r, the line bundle $L_1^{\otimes r} \otimes L_2$ is positive.

Solution:

This problem involved the kind of delicate analysis which we did not do in class, so your *homework* was to do some literature search and find it yourself.

Finding the relation between positiveness and ampleness is one way of doing it and I accepted that solution.

More in line with the spirit of what we are doing in class would be to calculate it directly. The idea is to see that if Θ_i is the curvature matrix of L_i, then $r\Theta_1 + \Theta_2$ is the curvature matrix of $L_1^{\otimes r} \otimes L_2$. Then using some compactness arguments you can conclude that, roughly speaking, Θ_1 and Θ_2 each has a minimum on M, and since the minimum of Θ_1 is positive, r can be chosen to make $r\Theta_1 + \Theta_2$ positive. A good place to follow how these ideas are put into explicit expressions I refer you to Huybrechts' book, Section 5.2, pages 239-246.