Due Date: 4 January 2018, Thursday

Due Time: 17:00

NAME:..

STUDENT NO:.......................................

Math 503 Complex Analysis - Final Exam

1	2	3	4	TOTAL
25	25	25	25	100

Please do not write anything inside the above boxes!

Check that there are **4** questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.
Submit your solutions on this booklet only. Use extra pages if necessary.

General Rules for Take-Home Assignments

(1) You may discuss the problems with your classmates or with me but it is absolutely mandatory that you **write your answers alone**.

(2) You must obey the usual rules of attribution: all sources you use must be explicitly cited in such a manner that the source is easily retrieved with your citation. This includes any ideas you borrowed from your friends. (It is a good thing to borrow ideas from friends but it is a bad thing not to acknowledge their contribution!)

(3) Even if you find a solution online, you must rewrite it in your own narration, fill in the blanks if any, making sure that you **exhibit your total understanding of the ideas involved**.

Affidavit of compliance with the above rules: I affirm that I have complied with the above rules in preparing this submitted work.

Please sign here:

NAME: STUDENT NO: DEPARTMENT:

Q-1) Let $\eta(z) = \dfrac{\zeta'(z)}{\zeta(z)}$ for $\operatorname{Re} z > 1$, where ζ is the Riemann zeta function. Show that $\lim_{z \to z_0} (z - z_0)\eta(z)$ is always an integer for $\operatorname{Re} z_0 \geq 1$. What is this integer? Make sure you cover all the cases of $\operatorname{Re} z_0 \geq 1$.

Solution:

NAME: STUDENT NO: DEPARTMENT:

Q-2) Let ϕ be an analytic function on $\operatorname{Re} z > 0$, and satisfy the conditions

(a) $\phi(1) = 2017$,

(b) $\phi(z+1) = z\phi(z)$,

(c) $\displaystyle\lim_{n\to\infty} \frac{\phi(z+n)}{n^z \phi(n)} = 2018.$

Find $\displaystyle\lim_{z\to\pi} \frac{\phi(z)}{\Gamma(z)}$.

Solution:

NAME: STUDENT NO: DEPARTMENT:

Q-3 Show that

$$\sinh \pi z = \pi z \prod_{n=1}^{\infty} \left(1 + \frac{z^2}{n^2}\right).$$

Solution:

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Q-4) For any positive integer n, calculate

$$I_n = \int_0^{\infty} \frac{dx}{1 + x^{2n+1}}.$$

Solution: