Due Date: 12 December 2017, Tuesday

NAME:..

STUDENT NO:.......................................

Math 503 Complex Analysis - Midterm 2

1	2	3	4	TOTAL
25	25	25	25	100

Please do not write anything inside the above boxes!

Check that there are **4** questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.
Submit your solutions on this booklet only. Use extra pages if necessary.

General Rules for Take-Home Assignments

(1) You may discuss the problems with your classmates or with me but it is absolutely mandatory that you **write your answers alone**.

(2) You must obey the usual rules of attribution: all sources you use must be explicitly cited in such a manner that the source is easily retrieved with your citation. This includes any ideas you borrowed from your friends. (It is a good thing to borrow ideas from friends but it is a bad thing not to acknowledge their contribution!)

(3) Even if you find a solution online, you must rewrite it in your own narration, fill in the blanks if any, making sure that you **exhibit your total understanding of the ideas involved**.

Affidavit of compliance with the above rules: I affirm that I have complied with the above rules in preparing this submitted work.

Please sign here:

NAME: STUDENT NO: DEPARTMENT:

Q-1) The original proof of the Riemann Mapping Theorem assumes that the proper, connected and simply connected open set U is bounded. Show that this causes no loss of generality.
(Of course you cannot use the Riemann Mapping Theorem here!)

Solution:

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Q-2) On the internet find the original proof Riemann gave for his mapping theorem and explain the steps of the proof in your own words.

Solution:

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Q-3 Let G be a simply connected region which is not the whole plane and suppose that $\bar{z} \in G$ whenever $z \in G$. Let $a \in G \cap \mathbb{R}$ and suppose that $f\colon G \to D = \{z\colon |z| < 1\}$ is a one-to-one analytic function with $f(a) = 0$, $f'(a) > 0$ and $f(G) = D$. Let $G_+ = \{z \in G\colon \operatorname{Im} z > 0\}$. Show that $f(G_+)$ must lie entirely above or entirely below the real axis.
(There are solutions of this on the Internet. Again use your own wording in your solution in a way to show your understanding.)

Solution:

NAME: STUDENT NO: DEPARTMENT:

Q-4) Let $I_n = \displaystyle\int_0^\infty \frac{\log x}{(1+x^2)^n}\, dx$, $n = 2, 3, \ldots$.

Find a formula in terms of residues for I_n and using a software to calculate these residues, write the values of I_n for $n = 2, \ldots, 10$. *(Check privately using the same software that the values of I_n match your residue calculations.)*

Solution: