



**Prof. Dr. M. G. IKEDA
(1926 - 2003)**

Professor Masatoshi Gündüz Ikeda: A Life Devoted to Mathematics

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In the summer of 1963 as a new graduate I paid a visit to the newly established Mathematics Department of Ege University, in İzmir, to apply for a post. In the very moments of my arrival, they introduced to me a young faculty member: Doctor Ikeda. Apparently he was Japanese, polite yet authoritative, and there was an air of general admiration about him; however the rest was a complete mystery for me. I complied with his suggestions and brought a few more documents to support my application, and thus completed the application process and joined the department. My first duty there was to assist his lectures. However, this was not as easy as I had expected. One of his two courses was totally unfamiliar to me; the other one in turn was a course of which I covered much less as an undergraduate than had been taught by Dr. Ikeda. My ability to keep up with the subject was slowed by the fact that the books in Turkish or French I had access to were too few, causing me to consult desperately all the existing books thoroughly. The professor gave no credit for the cliché “az olsun ama tam olsun” (let it be little but complete) which is commonly used by lecturers as an excuse for covering limited teaching material. On the contrary he would always prefer to cover many diverse concepts and techniques with their essential features. Naturally there were students who are not able to grasp and appreciate this attitude. I remember a student replying to an exam question in which the students were to “give outlines of the proof of the fact that a general polynomial of degree greater than or equal to five cannot be solved by radicals” with the answer “God is so great that he will allow someone in the future to make it possible.” His lectures included neither unnecessary material, nor unnecessary ornaments and digressions. He was meticulous about the content and rigorous in his lectures. On

a day in 1968, vexed by the students movements, I remember him reprimanding his students, “I am not supposed to lecture colored, Turkish, nor cinemascope,” referring to the many cinema advertisements that had been appearing everywhere then. Not to be misunderstood, I must emphasize that he had neither difficulty in lecturing nor in writing official letters and reports in Turkish. Further, in later years I also witnessed him inserting Turkish words into their Japanese conversation with his Japanese friends. In a conference he said that the language in which he could express himself best was Turkish, even though he could fluently speak and write English, German and French—other than his native language.

Having been newly established, Ege University was short of facilities, resulting in, among other things, a limited number of offices in the Mathematics Department. Thus we assistants found ourselves sharing one single office. However, in time, thanks to Dr. Ikeda, we transformed this office to an ongoing seminar room, for we had problems in different areas of mathematics and Dr Ikeda was the only one to lend us a hand so benevolently. His generosity and kindheartedness would touch all the faculty at Ege University, as well as all the mathematics assistants, in all of their diverse areas, supervised by himself or not. He was even often found to be helping many a faculty applicant to associate professorship or professorship in Medicine or Agriculture in their various correlation problems. This reminds me of one of his witty stories. On a day he was very busy with his own application for associate professorship, an applicant from another faculty comes and asks for help about some statistical problems of his thesis. Dr. Ikeda apologizes and tries to explain in a polite manner how busy he is with his own associate professorship application and has no time to help him. Yet the visitor insists and says, “but your application is for associate professorship where as mine is for professorship; so mine is more important than yours.”

In 1964 many important events came to pass: he married to Emel Ardor, received Turkish citizenship, changed his name as Masatoshi Gündüz İkedâ and, in a relatively short time, was promoted to associate professorship and then to professorship.

Masatoshi Ikeda was born on the 25th of February, 1926, in Tokyo as the youngest child of a statistician father and a teacher-mother after two daughters and a son, who passed away in his 20's. He graduated from Osaka University in 1948, receiving the Rigaku-Shi (B.S.) degree. The same year he took a post at Osaka University and started to study Frobenius and quasi-Frobenius algebras under the supervision of K.

Shoda. It was there he encountered a working group of researchers led by T. Nakayama, and joined them. He published his first paper [1] jointly with T. Nakayama, that characterized Frobenius algebras, and it had been followed by [2] and [3] giving new characterizations of quasi-Frobenius algebras in terms of Shoda's self injectivity condition. In [4] he generalized a commutativity theorem due to Kaplansky; his main result there was the semi-primitive case of the well-known commutativity theorem given later on by N. Jacobson. In 1954 he published [5], generalizing Gaschütz's results on group algebras to Frobenius rings. The operator introduced there is known as the Gaschütz-Ikeda operator and in symmetric algebras the image of this operator turns out to be an invariant of the algebra. These papers, together with [6] and [7], constituted his doctoral dissertation and he received the Rigaku-Hakushi (Ph.D.) degree from Osaka University in 1953. His joint paper [6] with Nakayama gives some characteristic properties of quasi-Frobenius rings in terms of annihilators of intersections and it constitutes the source of Ikeda-Nakayama rings introduced in [43] and Ikeda-Nakayama modules in [44]. In a group including T. Nakayama, H. Nagao and later S. Eilenberg, he published a sequence of papers [8, 11, 12] on the cohomology of algebras. One might say his ring theory period continued until 1955.

The Algebraic Number Theory Symposium held in Tokyo in 1955 became one of the turning points of his life, for he then turned his interest to number theory and decided to join Professor Hasse's school at Hamburg University. Between 1957 and 1959 he continued his research at Hamburg University as an Alexander von Humboldt Foundation fellow. In this period he concentrated on the embedding problem of Galois theory and he obtained an important reduction on the existence of strong solutions of a finite embedding problem with abelian kernel. The Alexander von Humboldt Foundation was the second influential item in his life, for besides its fellowship which allowed him to join Hasse's school, its social activities gave rise to meeting Dr. Emel Ardor, who was also a fellow of the foundation, thus further precipitating his interest in Turkey. Dr. Emel (Ardor) Ikeda recalls his curiosity: "Dr. Ikeda began to show me his interest. He asked me several questions about Turkey, Turkish, life and customs in Turkey. I soon found out the reason for this curiosity. His teacher Professor Hasse had visited Turkey a couple of times, he loved Turkey, he had friends and students in Turkey and he was keen on learning Turkish. Further, he met Prof. Orhan İçen in Hamburg, he knew Cahit Arf by name.

[Ikeda] was keen on learning new things; [and] he, too, developed a passion for Turkey and Turkish.” (See [45].) This passion brought him to İzmir, Turkey in 1960. Soon after, he visited İstanbul University to meet Ord. Prof. Dr. Cahit Arf, who was a former doctoral student of Professor Hasse. This was the beginning of an eternal friendship and a lifetime in Turkey. For a short while he taught statistics at the Medical Faculty of Ege University. In 1961 he joined the newly established Mathematics Department of the Science Faculty where he shouldered all the academic and administrative duties and loads alone as the head of the Chair of Theoretical Mathematics. Meanwhile, he paid varied visits to Hamburg University, Middle East Technical University in Ankara, and Turkish Scientific and Technical Research Council (TÜBİTAK) presided by Professor Arf to keep in touch with other colleagues with who he could collaborate. On the last days of May 1968, Professors C. Arf and R. Laglands visited Ege university to deliver talks on the Cartan-Dieudonné Theorem and Automorphic Forms, respectively. In the May 31st Karşıyaka dinner following the talks (also mentioned in an article by Professor Laglands (see [46]), Ikeda was invited to join METU. Another important aspect of this visit is revealed in this article. May it be known that the seeds of Lagland’s Program had been planted in a hotel in Konak, İzmir during this visit.

Up to the end of his stay in İzmir, the Professor continued his research on the embedding problem, focusing on Grunwald’s Existence theorem. The work appeared in [17] and [18]. In addition, he published a paper [16] on the crossed products of S-fields and he answered a question on Turan inequalities raised by Professor Kochmieder in one departmental seminar in his paper [19]. The papers [20] and [21], which deal with noncommutative rings, are also products of these years.

In August 1968 Ikeda moved to Ankara and joined Middle East Technical University, as expected. Together with Professor Arf, he took on the mission of establishing a genuine mathematics department, a genuine graduate school of mathematics, with the hope of developing a genuine center of mathematics, where mathematicians educated at METU would take responsibility for establishing new universities and institutions in other cities of the country. Part of this dream has come true, even thought in a different, unforeseen form. I am convinced that that the mission of converting this dream into reality, in its every detail, is a most vital legacy and testament to this inseparable pair of mentors. In the late 60’s and 70’s, the most prominent formation was the TÜBİTAK

Pure Mathematics Unit, presided by Professor Ikeda, although it may be remembered by only a very few today. The aim of this unit was to unite all mathematicians at METU, as well as in Ankara and around the country, and those completing their Ph.D.'s abroad, around the world, into one single entity, to activate them and thus create a mathematical community, speaking almost the same mathematical language. I feel that this goal has been achieved in a considerable extent. In this unit every member prepared his own semi-annual project, progressed his or her solutions to problems in the project step-by-step in relatively small seminars and then presented results obtained in weekly departmental seminars. In the 1970's, the Silivri Research Institute (Silivri Araştırma Enstitüsü) conferences were added to the pursuits of the unit. This Institute was founded by the late Professor Nazım Terzioğlu who was then the Rector of Istanbul University, and it was the place to gather for two or three international conferences each year. During the period, Professor Ikeda took on the mission of motivating, encouraging and giving suggestions to the members of the unit, while continuing with his own individual and joint problems, on which he obtained remarkable results. He worked on diverse areas of algebra such as algebraic geometry, algebraic number theory, the theory of rings, algebras and modules, including also non-Archimedean normed spaces and C^* -algebras. At METU, Professor Ikeda had many advantages compared with his previous years in Turkey. He found colleagues to share academic responsibilities, he had no obligation to take on administrative duties except for the reasonable bureaucracy of the Pure Math Research Unit, there were graduate courses and a graduate school concept to improve, and yet there was a rich library open 27/7 and a well-stocked and active bookstore, etc. Publications of this fruitful period includes [22–36] of which I believe [33] was rather special to him. In this paper he proved a conjecture due to Jürgen Neukirch, who was one of Krull's students and one of the most talented algebra and number theorists of the world, passed away at an early age. At a meeting at Oberwolfach, Professor Neukirch mentioned his conjecture that continuous group automorphisms of the Galois group of the separable closure of the rational number field probably consists of inner automorphisms. This attracted the Professor's attention a lot. He first proved in [31] that every continuous group automorphism of the Galois group of the separable closure (or of the solvable closure) of the rational number field leaves each conjugate class of the Galois group and then he succeeded in [33] to complete the proof of the conjecture. Professor Neukirch

regarded Professor Ikeda as his teacher since he impressed and inspired to a large extent by the Professor's previous works. Being able to conclude affirmatively his "student's" perfectly formulated striking conjecture pleased the Professor very much. The same conjecture had been proved also independently by Iwasawa and Uchida at the same time. We enjoyed hearing Professor Neukirch's talk on his own conjecture entitled "The Ikeda-Iwasawa-Uchida Theorem" at a symposium in Silivri in 1975. This symposium, whose participants include Hasse, Arf, Roquette, Neukirch, İçen, Frey and Geyer, was an important, historical symposium held in Turkey. The unique and natural organizer of such symposiums on algebra and number theory was Professor Ikeda in collaboration with Professor Arf. Mathematical pursuits of this sort took place at the Nazım Terzioğlu Research Institute in Silivri through to the 1980's. After a one year stay at the Institute for Advanced Study at Princeton in 1976, and a short stay at Hacettepe University, he returned again to METU in 1978. No one could expect any favor for mathematics from the hard times of the 1980's. He visited Yarmouk University in Jordan twice; and in between the two visits, his colleagues at METU requested him to hold the chair of the department. It was a nightmare, but there was nothing to do but to lend a hand. Between 1983 and 1985 the Professor's presence was the only encouragement for the department. Aside from this, he spent most of decade of the 80's visiting Yarmouk University and Eastern Mediterranean University in North Cyprus. In 1992 he retired and joined Eastern Mediterranean University. His main interest in Jordan and Cyprus was the higher order derivations, which resulted in the publication of [35], and the homological aspects of algebras. He worked on generalizing differential modules due to Y. Nakai as Universal Modules with respect to n -cocycles. He also worked on integral domains whose results appeared in [36] and [37]. In 1993 he was invited by Marmara Research Center of TÜBİTAK in Gebze, İstanbul, which had been founded by a group of scientists including Professor Ikeda, with the hard work of nearly a decade. In the center he headed a small research team who concentrated on cryptography and related topics and published three papers [38], [39] and [40]. In 1997, he moved to the newly established Feza Gürsey Research Institute in Kandili, İstanbul, to concentrate on his number theory problems primarily. Part of the outcome of his work appeared in [41].

Professor Ikeda received the most prestigious awards of Turkey for his outstanding contribution to mathematics. The first of them is the Science Award of TÜBİTAK, in

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1978. In 1994, he was awarded the Mustafa Parlar Honorary Science Award, followed soon after in 1996 by the Merit Award given by the TÜBİTAK Marmara Research Center. He had been a member of the TÜBİTAK Fundamental Sciences Group and on the Advisory Board of the Turkish Journal of Mathematics for many years. He was elected to the Turkish Academy of Sciences in 1997. The pride of his distinctions must be the long list of his students; yet we are able to give only a small part of this list. No doubt, the actual list of formal or informal students who have profoundly benefited from his suggestions and advice in their theses may be uncountable. His students include Timur Karaçay, Cemal Koç, Birol Temelkuran, Arif Kaya, Nurettin Cengiz, Gönül Uslu, Bekir Kılıçoğlu, Mahmut Hayfavi, Abdullah Harmanlı, Sadi Abou Saymeh, Rüstem Kaya, Ahmet Sharary, Abdurrahim Yılmaz, Fethi Çallıalp, Cesarettin Koç, Mustafa Akgül, İbrahim İbrahimoglu, Hasan İlhan Tutalar, James Conlon.

He was so kindhearted and benevolent that probably there was very few who met him that did not receive profound suggestions and advices to improve his/her work and ideas. In some gatherings, participants often urged him to talk on mathematics and mathematicians in general. I remember two such occurrences, one held in Altınoluk in a symposium organized by Balıkesir University, and the other at METU. They were full of panoramas of mathematics, wisdom refined in his extensive experience and deep knowledge, witty jokes, and indispensable warnings and advice, always valid. Who can forget his voice at Altınoluk saying, “Avoid repeating yourselves in order to increase the number of publications only. Work out real problems of mathematics no matter how big they are”; or yet his unforgettable advice “When you worked on a problem for along time without any outcome, leave it alone for a while and consider also some relatively small problems to keep up your courage and productivity.” His memories of the difficult days of the Second World War in Japan may be a light of hope to every youngster in despair. He once recalled, “We were out of food, at the edge of starving, yet we were reading Pontriagin’s Topological Groups without skipping a word.” Yet his suggestion and warning in [34] to young mathematicians about the works of Arf (now equally valid for his own works): “To my regret, however, Cahit Arf never had pupils (in a true sense) in Turkey; it might be because he is too great, or because his works are too hard for common mathematicians. In any case, it is a pity not for Cahit Arf but for Turkish Mathematics. I really do not see why Arf rings are studied by American

mathematicians yet not by Turkish mathematicians, and why his wonderful thesis is intensively re-examined by German mathematicians yet not by Turkish mathematicians. I am sure that the growth of Mathematics in a country is, as history shows, only possible if the mathematicians in that country mathematically understand and stimulate each other. So I should like to emphasize again that it is the task of young Turkish mathematicians work in these fields, first to learn by heart what Cahit Arf did, and then to continue further study along the lines indicated by him.”

At the beginning of 2002, he began to dream again, of gathering young Istanbul researchers around a pursuit that might motivate and lead them to better collaboration. In this direction, beginning February 2002, he organized weekly seminars at the Karaköy communication center of Sabancı University. He faithfully attended these seminars until his illness came out in the summer of 2003. He delivered most of the lectures and supplemented and completed other lectures by his invaluable comments and remarks that we will always remember and sorely miss. 10 February 2003 might be said to be the QED at the end of his life, a life devoted to mathematics. Any word after this point would be superfluous, but it is a debt for me to express on behalf of myself, his students, his colleagues and all the mathematicians of the country, our praise, admiration, esteem and our deep gratitude to him.

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