

BILKENT UNIVERSITY
Department of Economics
Ec453 Theories of Growth and Development I
Fall 2008 Final Exam

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1. (25 points) Suppose that in a given economy single consumption good, Y , is produced under competitive conditions. The technology is given by the following function: $y = f(k) = Ak$, where y and k are output and capital per person, respectively; and A is a strictly positive parameter depicting the state of technology.

The representative consumer maximizes the following felicity function: $U(c) = \int e^{-\rho t} \left[\frac{c_t^{1-\theta} - 1}{1-\theta} \right] dt$. c_t is consumption per person, and ρ is the subjective discount rate. Assume further that population growth rate is zero in this economy. The consumer's intertemporal budget constraint satisfies that asset accumulation equals interest income net of consumption. Thus, $\dot{a} = ra - c$, where a stands for assets, and r is interest rate on asset holdings.

Law of motion of capital obeys: $\dot{k} = f(k) - c - \delta k$, where δ is the rate of depreciation.

a. assuming that the economy is closed to trade, solve the producer's and the consumer's intertemporal optimization problems. State the general equilibrium of the economy. In your description solve for the (endogenous) rate of growth of output and the law of motion of the optimal consumption profile. Portray the endogenous growth path of the economy within a graph.

b. Define the saving rate as the ratio of total investment to output, *i.e.*, $s = \frac{K + \delta K}{Y}$, where K is aggregate capital, and Y is aggregate output. Thus, $Y = AK$. State the value of the saving rate under long run equilibrium growth as a function of the technology level, depreciation rate, subjective discount rate and other parameters of the model. How is saving rate affected to changes in ρ ? Discuss and interpret your result.

2. (20 points) Suppose that in the country of Solovia output is produced along the production function: $Y = K^\alpha L^{(1-\alpha)}$. where Y is output, and K and L are the inputs of capital and labor, respectively. Suppose that the growth rate of the labor force is zero, and the rate of depreciation of capital is a known value, δ . Solovians are currently experiencing steady state growth where their per capita consumption had been maximized at the golden rule.

- a. Show the equilibrium position of per capita output in the long run steady state, as a function of capital per labor. In your graph clearly mark the level of consumption per capita, the wage rate, and total profits per labor. What do you know about the saving rate and the share of capital income in total output for this economy under the steady state equilibrium? (No proofs are necessary, simple illustration and mentioning key relationships suffice).
- b. Now suppose that due to a shift in job preferences of the young generation, Solovia started to suffer from natural unemployment, at the rate u . So the amount of available labor employed becomes $(1-u)L$, rather than L as above. Show in your graph how the steady state will be affected. Clearly follow the implications of this outcome on consumption, profits and wages per labor.

3. (25 points) Consider the R&D-driven endogenous growth model with the following technology for the production of the final good: $Y = L_Y^{(1-\alpha)} \sum x_i^\alpha$.

a) Inputs x_i are produced by oligopolistic firms given demand from the final good producer. Suppose that intermediate input producers hire other inputs to produce a new input at the rate r . For simplicity, assume that the input output coefficient for producing one unit of x is 1. Draw graphically the equilibrium of the oligopolist in the price-output space and interpret the optimal pricing rule of the oligopolist.

b) Using the fact that aggregate “capital” is $K = \sum x_i = Ax$ Show that the above technology is equivalent to: $Y = K^\alpha (AL_Y)^{(1-\alpha)}$

c) Show that payments to L_Y and total x (that is, capital K) in terms of w and r do not exhaust total value of the final output. Discuss what happens to the difference.

d) Suppose that the government introduces a subsidy on the wages of researchers (and of the university professors!). Suppose that the subsidy is limited only to the researchers, and that workers in the final goods-sector do not enjoy the subsidy. Discuss the implications of this policy on the R&D production, the production and price of intermediate inputs, and on the rate of growth of the economy. Is it possible for the growth rate under government’s intervention to exceed the rate of growth that can be achieved under the market equilibrium? Discuss.

4 (20 points) Keynesia is a small, private ownership economy. There is only one homogenous good, wheat, which is produced using labor and seeds of wheat alone. Land is in abundant supply, and is not considered to be a scarce good.

Currently the following data is being observed in Keynesia: output per labor, q , is 2.5; investment per labor, i , is 1; and consumption per labor, c , is 1.5. The technology in use in Keynesia's wheat production admits that the ongoing capital-labor ratio, k , is 0.5.

Consider the classical (i.e., Keynesian-neo-Ricardian and neo-Marxian) characterization of the Keynesian economy where (i) workers' wages are driven to subsistence consumption; (ii) workers do not save; and (iii) capitalists do not consume. Under these conditions,

- a) State the capital output ratio, v ; and the labor output ratio, l .
- b) Given that the wage rate (wages per labor) is equal to consumption per labor, find the profit rate in Keynesia. Verify the unit cost price equation: $1 = wl + (1 + r)v$; and the market equilibrium equation: $1 = cl + (1 + g)v$.
- c) Now suppose that the investment demand function is given by $g = i(r) = r^2$. Using information you derived thus far, find the prevailing growth rate in Keynesia. Check the macro equilibrium using the demand-supply balance. Portray the Keynesian-Neo-Ricardian equilibrium of this economy in a 4-quadrant graph of w , r , g , and c .
- d) How does the Keynesian equilibrium configuration differ conceptually from the Marxian description of the same equilibrium?

5 (10 points) One of the distinguishing features of the *Schumpeterian* growth from standard mainstream treatments of innovation-driven endogenous growth is the notion of *creative destruction*.

- a) Briefly describe what you understand from the phrase *creative destruction*.
- b) Denoting output by X , the oligopolistic mark up by μ , and the period during which the product will be operational by D , write the profit function under the Schumpeterian equilibrium.
- c) Let δ denote the productivity of researchers. Assuming that innovation is produced by labor alone, write the average cost function for innovation in terms of labor costs and productivity.
- d) Suppose that supply of total resources is given by R , and innovation requires a total of N resources in this economy, with $N = nR$. The expected number of innovations is given by $E[A] = \delta N$, and expected time of operation is $E[D] = \frac{1}{E[A]}$. Write the expected profit function and the present value of innovations for this economy.
- e) Suppose that due to a sudden change of sentiment, researchers started to submit themselves more to the joys of life rather than devoting their energies to pure sciences. As a consequence, δ is observed to fall. Discuss the effects of this development on ATC, PVI and N. Use graphs if necessary.