

BILKENT UNIVERSITY
Department of Economics
Ec453 Theories of Growth and Development I
Fall 2009 Final Exam

Professor Erinc Yeldan

1. Consider the neoclassical (Solow) model of exogenous growth with the following technological set up: Single output Y is produced by labor, L , and capital, K . The production technology is:

$$Y = AK^\alpha L^{1-\alpha}$$

where there is no technological progress, thus A is constant. Labor grows at the rate n , and capital depreciates at δ . A fraction s of output is saved and invested for capital accumulation.

- (a) Find an expression for the output per worker Y/L under the steady state.
- (b) What is the rate of growth in *aggregate* GDP in this economy under the steady state? What is the rate of growth of *per capita* GDP under the steady state?
- (c) *True or False*: Suppose that the saving rate s is increased permanently. Then output per worker is increased permanently under the long run steady state equilibrium.
- (d) *True or False*: A permanent increase in the saving rate increases consumption per worker under the steady state.
- (e) Suppose that due to outmigration and fertility problems, the population growth rate is cut by half. How will the transition and the steady state be affected? If you think that the steady state of the economy will be changed, will the per capita output be higher or lower under the new steady state?

2. Suppose that in a given economy single consumption good, Y , is produced under competitive conditions. The technology is given by the following function: $y = f(k) = Ak^\alpha$, where y and k are output and capital per person, respectively; and A is a strictly positive parameter depicting the state of technology. The representative consumer maximizes the following felicity function: $U(c) = \int e^{-\rho t} \left[\frac{c_t^{1-\theta} - 1}{1-\theta} \right] dt$. Here, c_t is consumption per person, and ρ is the subjective discount rate. Assume further that population growth rate is zero in this economy. The consumer's intertemporal budget constraint satisfies that asset accumulation equals total net income minus consumption. Thus, $\dot{k} = f(k) - \delta k - c$, where δ is the rate of depreciation.

- (a) Assuming that the economy is closed to trade, solve the consumer's intertemporal optimization problem. State the general equilibrium of the economy. In your description solve for the (transitional) rate of growth of output and the law of motion of the optimal consumption profile. Portray the steady state of the economy within a graph. Is the steady state equilibrium unique? Is it stable?
- (b) Suppose that the government decides to tax consumption at the rate t . Show how the steady state equilibrium is affected.
- (c) Now suppose that the government decided to tax output at the same rate. Contrast the effects of the output tax with the tax rule in (b) above. Which tax regime would you prefer as an economist?

3. Consider the R&D-driven endogenous growth model with the following technology for the production of the final good: $Y = L_Y^{(1-\alpha)} \sum x_i^\alpha$. Inputs x_i are produced by oligopolistic firms given demand from the final good producer. Suppose that intermediate input producers hire other inputs to produce a new input at the rate r . For simplicity, assume that the input output coefficient for producing one unit of x is 1. Draw graphically the equilibrium of the oligopolist in the price-output space and interpret the optimal pricing rule of the oligopolist.

- (a) Suppose that the R&D function in this economy is *Romerian*, *i.e.*

$$\dot{A} = \gamma L_A A$$

Suppose that the government introduces a subsidy on the wages of researchers (and of the university professors!). Suppose that the subsidy is limited only to the researchers, and that workers in the final goods-sector do not enjoy the subsidy. Discuss the implications of this policy on the R&D production, the production and price of intermediate inputs, and on the rate of growth of the economy. Is it possible for the growth rate under government's intervention to exceed the rate of growth that can be achieved under the market equilibrium? Discuss.

- (b) Now suppose that the R&D function is *Jonesian*:

$$\dot{A} = \gamma L_A^\lambda A^\phi$$

where $0 < \lambda, \phi < 1$. Solve the above problem (b) under the *Jonesian* framework. No formal proofs are necessary; an intelligent analytical approach is all that is needed.

4. *Keynesia* is a small, private ownership economy. There is only one homogenous good, wheat, which is produced using labor and seeds of wheat alone. Land is in abundant supply, and is not considered to be a scarce good. Currently the following data is being observed in Keynesia: output per labor, q , is 2.5; investment per labor, i , is 1; and consumption per labor, c , is 1.5. The technology in use in Keynesia's wheat production admits that the ongoing capital-labor ratio, k , is 0.5

Consider the classical (i.e., Keynesian-neo-Ricardian and neo-Marxian) characterization of the Keynesian economy where (i) workers' wages are driven to subsistence consumption; (ii) workers do not save; and (iii) capitalists do not consume. Under these conditions,

- (a) State the capital output ratio, v ; and the labor output ratio, l .
 - (b) Given that the wage rate (wages per labor) is equal to consumption per labor, find the profit rate in Keynesia. Verify the unit cost price equation: $1 = wl + (1 + r)v$; and the market equilibrium equation: $1 = cl + (1 + g)v$.
 - (c) Now suppose that the investment demand function is given by $g = i(r) = r^2$. Using information you derived thus far, find the prevailing growth rate in Keynesia. Check the macro equilibrium using the demand-supply balance. Portray the Keynesian-Neo-Ricardian equilibrium of this economy in a 4-quadrant graph of w , r , g , and c .
 - (d) How does the Keynesian equilibrium configuration differ conceptually from the Marxian description of the same equilibrium?
5. One of the distinguishing features of the *Schumpeterian* growth from standard mainstream treatments of innovation-driven endogenous growth is the notion of *creative destruction*.
- (a) Briefly describe what you understand from the phrase *creative destruction*.
 - (b) Denoting output by X , the oligopolistic mark up by μ , and the period during which the product will be operational by D , write the profit function under the Schumpeterian equilibrium.
 - (c) Let φ denote the productivity of researchers. Assuming that innovation is produced by labor alone, write the average cost function for innovation in terms of labor costs and productivity.
 - (d) Suppose that supply of total resources is given by R , and innovation requires a total of N resources in this economy, with $N = nR$. The expected number of innovations is given by $E[A] = \varphi N$, and expected time of operation is $E[D] = \frac{1}{E[A]}$. Write the expected profit function and the present value of innovations for this economy.
 - (e) Suppose that due to a sudden change of sentiment, researchers started to submit themselves more to the joys of life rather than devoting their energies to pure sciences. As a consequence, φ is observed to fall. Discuss the effects of this development on ATC , PVI and N . Use graphs if necessary.