

**Ec453 Theories of Growth and Development**  
**Fall 2001, Midterm I**

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YOU HAVE 3 HOURS FOR THIS EXAM. THUS TIME IS AN EXTREMELY SCARCE GOOD. USE IT **OPTIMALLY**

1) (5 points) Discuss analytically as an economist what you understand from the phrase above, that “time is scarce and you have to allocate it **optimally**”.

2) (30 points)

Consider the following private ownership economy: There is only one homogenous good, wheat, which is produced using labor and seeds of wheat alone.

The total wheat harvested is given by the (neoclassical) production function  $q = k^\gamma$ . Currently 0.1 units of wheat is saved and invested as seeds for next year’s production cycle.

The population growth rate is zero in this economy. Bear in mind that in this model, seeds invested in the previous year become capital stock of the current year. Otherwise capital cannot be stored and accumulated. Thus depreciation of capital,  $\delta$  is 1.0. The technology parameter,  $\gamma$ , is known to be 0.5.

a) Solve for the neoclassical long run equilibrium growth (the steady state equilibrium) values of the capital labor ratio, output per labor in this economy. Solve for the gross profit rate  $(r + \delta)$  and the wage rate under the neoclassical steady state equilibrium?

(i) Verify that the unit price equation holds for this economy:  $1 = wl + (r + \delta)k_y$

(ii) Using the classical saving function obtain the Marxian growth rate for this economy.

(iii) Using the demand-supply equilibrium ( $1 = cl + gk_y$ ) find the per capita consumption level in the Marxian model. Show the Marxian equilibrium in a 4-

- quadrant graph, using the wage rate, consumption, growth rate and the gross profit rate values you have depicted above
- (iv) Now find the per capita consumption level in the neoclassical model under the steady state. Show it with the aid of a graph on the  $y - k$  space depicting the neoclassical steady state equilibrium.
- (v) Contrast the neoclassical and the Marxian values of per capita consumption. How do you account for the differences?
- b) Now consider the *Golden Rule of Accumulation* under neoclassical model. Find the new saving rate that would maximize per capita consumption per capita consumption under the steady state. Find neoclassical wage rate and the *net* profit rate ( $r$ ) under the golden rule.
- (c) Find the capital labor ratio and the wage rate and the net profit rate under the *Golden Age of Capital* that is the value of  $k$  that would maximize the net profit rate in this economy.
- (d) Show the neoclassical steady state equilibrium values of the golden rule and the golden age. Is the capital labor ratio higher or lower in the Golden Age as compared to the Golden Rule? How do you account for this difference?

### 3) *Investing in Capital* (15 points)

Consider the following three countries, A, B, and C. Each country is endowed with the same amount of capital and labor, and the respective saving rates out of national product are the same. Suppose that the countries are also identical with respect to their technologies in producing the final good and they are initially at their initial steady states. Finally assume that the rate of population growth is the same across countries, and that there is no (exogenous) technical change occurring.

Suppose now that in each country a dictator assumes power and announces the following policies: *Country A* announces that it will increase its saving rate by a factor of two for the next ten years to “provide better living conditions for the next generations and to surpass the other two countries”. It plans to use its extra savings on investing physical capital. After the end of the ten-year policy the saving rate is planned to be set to its initial rate. *Country B* announces that it will also increase its saving rate by a factor of two for the next ten years to “provide better living conditions for the next generations and to surpass the other two countries”; and plans to invest on education and human capital formation –*i.e.* producing “educated labor force and knowledge”. After the end of the ten-

year policy the saving rate is planned to be set to its initial rate, just like in *A*. *Country C* does not follow any change in its existing economic policies.

Discuss the nature of the transitional growth after the dictators assume power in all three countries. What will be the per capita rates and levels of growth in the respective countries at the new steady states? Discuss the position of the new steady states in comparison to the initial conditions. Use graphs or algebraic formulas, if necessary.

4) (10 points)

Consider the kaldorian model where workers do not save, and investment is given from outside with  $I = I^*$ . Show the share of wages in output and verify that it is inversely related with the Investment share of aggregate demand.

5) (15 points)

- a) Suppose that an economy obeying neoclassical assumptions was experiencing transitional dynamics towards steady state. Calculate analytically, the rate of growth of output per labor in this economy.
- b) Now suppose that in “midway” to the steady state an earthquake occurred and a portion of capital; stock is destroyed. How is the economy’s steady state and transitional dynamic growth affected? Is the new rate of growth faster or slower in comparison to the pre-earth quake situation?

6) (25 points)

Consider the following economy. Assume that per capita output,  $y$ , is given by the Cobb-Douglas function, with

$$y = f(k, g) = k^\alpha g^{1-\alpha}$$

where  $k$  is the capital labor ratio and  $g$  is government spending per labor.

Labor is increasing in supply at the rate of  $n$ ; and there is no technological growth. Investment per labor accumulates given the law of motion:

$$\frac{I}{L} = s(1-t)f(k, g)$$

where  $f(k, g)$  is per capita production function,  $s$ , and  $t$  is the tax rate to finance government expenditures per capita,  $g$ . Government budget is in balance, thus, in per capita terms:

$$g = tf(k, g)$$

- a) Derive the rule for the rate of capital accumulation for this economy. What is the steady state growth rate of  $k$ ? Show the transition dynamics and the steady state in a graph.
- b) How does the steady state rate of growth of  $k$  depend on  $t$ ? Find the growth rate maximizing value of the tax rate,  $t$  in this economy.