Ec453 Theories of Growth and Development I
Fall 2003, Final Exam

Prof Erinç Yeldan

1) (40 points)
Consider the R&D-driven endogenous growth model with the following technology for the production of the final good:

$$Y = L_Y^{1-\alpha} \sum_i^A x_i^\alpha$$

a) Given the above technology, find the demand function for labor and the intermediate good, x_i.
b) Suppose that one unit of x_i is being produced by η units of the foregone output. Thus the profit function of the intermediate good producer can be written as:

$$\pi(x) = p_i(x)x_i - r\eta x_i$$

Find the optimal price in terms of a mark up and the marginal cost. What is the amount of the mark up? Find the optimal level of profits.

c) Draw graphically the equilibrium of the oligopolist in the price-output space and interpret the optimal pricing rule of the oligopolist.
d) Show that payments to L_Y and total inputs Ax in terms of w and r do not exhaust total value of the final output. Discuss what happens to the difference.
e) Suppose that the research sector produces blueprints (knowledge) with the following function:

$$\dot{A} = \delta s_R L$$

where s_R is the share of labor employed in research, and δ is a parameter indicating the productivity of the research workers. Let the growth rate of population be n, i.e. $\frac{\dot{L}}{L} = n$.

Show that knowledge output grows at n. ($\frac{\dot{A}}{A} = g_A = n$).

f) Interpret the above research equation in (e). How does it differ from Romer's original research function in his *1990 JPE* paper? What are the implications of these different functional forms for the causality of output growth? In what sense is growth *endogenous* in the model portrayed in part (e)?

f) Show that per capita output (Y/L) increases with s_R only for $0 < s_R < ½$. Thus, over-commitment to research may be Pareto inferior for this economy. Interpret this result.

2) (30 points)
Suppose that two countries, *Jonesia* and *Romeria* have the following research production functions, respectively:

Jonesia: $\dot{A}_J = \delta_J L_J^{3/2}$

Romeria: $\dot{A}_R = \delta_R L_R A_R^{1/2}$

With $\delta_R<\delta_J$ forever. Suppose that both countries have the same population growth rate, n=0.01.

a) Assume that initially Romeria is poorer, i.e., $y_R < y_J$. Given the productivity differences ($\delta_R<\delta_J$), would you expect that Romeria will remain poorer than Jonesia forever? Justify your result. In particular, which country grows faster?
b) What can you say about the *real wage rates* in research in Jonesia versus Romeria? Which country has higher real wages for the research personnel? How do real wage rates change over time?

3) (20 points)
Consider the Rebelo (1991) model of endogenous growth with $y = Ak$, where y is per capita output, and k is capital-labor ratio. Suppose that agents in this economy have the following representative intertemporal felicity function:

$$U = \int_{t=0}^{\infty} e^{-\rho t} \frac{c_t^{1-\sigma} - 1}{1-\sigma} dt$$

where $\rho>0$ is the subjective discount rate. Capita labor ratio evolves over time with: $\dot{k} = Ak - c$, where c is per capita consumption.

a) Using the *Hamiltonian* of this system which maximizes per capita consumption subject to the capital accumulation constraint, find the long run equilibrium rate of growth of per capita consumption for this economy.
b) Find the equilibrium rate of savings for this economy? How is the saving rate affected in changes of ρ and σ?

4) (10 points)
Given your assessment of the various theories of growth, write a brief essay on the sources of growth and the nature of differences in per capita income across countries. Given you reading of the relevant literature, what are your key criticisms of the growth state of growth modeling? Cite all relevant literature in support of your arguments.