

**Ec453 Theories of Growth and Development**  
**Fall 2003, Midterm**

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1) (10 points)

Consider the neoclassical production technology: Single output  $Y$  is produced by labor,  $L$ , and capital,  $K$  in the following form:

$$Y = K^\alpha L^{1-\alpha}$$

Suppose that capital does not depreciate.

Define the following expressions analytically (in algebraic form):

- a. The wage rate
- b. The rental rate on capital (profit rate)
- c. Total profits per labor

Show these expressions with the aid of a graph of the per capita production function against capital per labor.

2) (20 points)

Consider the standard neo-classical (Solowian) model with one good being produced by capital and labor, using a neoclassical production technology. Suppose that population growth rate is zero, and that the depreciation of capital is  $\delta > 0$ . Suppose that the economy is initially at steady state equilibrium.

Now suppose that due to rumors of an earthquake, the residents of this country suddenly reduced their average savings rate (permanently) by half, with no other change in the remaining parameters of the economy. Do you expect any change in the steady state equilibrium of this economy? If so, discuss the effects of this reduction on per capita output, the capital labor ratio, the wage rate, profits per labor, and the profit rate under the new steady state.

3) (20 points)

Consider the following Kaldorian economy:

Single output  $Y$  is being produced by labor and capital. Investment demand is given by the following function:

$$I = 0.10Y$$

It is known that workers' saving propensity out of their wage income is 0.05 and that of capitalists is 0.20. The economy is closed to foreign trade and there is no government.

- Find the share of profits in total income under equilibrium.
- Now suppose that the investment function is changed to:  $I = 0.15Y$ . How will the share of profits and wages be affected? Discuss the mechanics of your algebra analytically.
- What is the maximum ratio of investments to income,  $(I/Y)$  that this economy can sustain in the long run?
- Now suppose that the investment demand has been reduced to  $I=0.05$ . Discuss the behavior of this economy under the Kaldorian assumptions.

4) (30 points)

Consider the Cass-Koopmans-Ramsey model of intertemporal optimization. Suppose that the instantaneous felicity of the representative consumer is given by the following:

$$U(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

Suppose that population growth rate is zero, and that there is no technological growth. The budget constraint of the agent is

$$\dot{a} = w + ra - c$$

where  $a$  is assets per person,  $w$  is wage rate, and  $r$  is the rate of return rate on assets. Assume that aggregate output  $Y=F(K, L)$  is given by the Cobb-Douglas function, with

$$F(K_t, L_t) = K_t^\alpha L_t^{1-\alpha}$$

Assume that the economy is closed to foreign trade. In this economy, the consumer tries to choose an optimal consumption profile by maximizing the discounted utility derived from the stream of future consumption subject to the budget constraint. Assuming that consumers discount the future at the constant subjective discount rate,  $\rho$ , and that initial capital stock is given,

- Set the consumer's intertemporal optimization problem as a discounted stream of future consumption profile subject to the budget constraint. Explicitly state the Hamiltonian of the system.

b) Solve the consumer's optimization problem.

Hint: The first order conditions of the discounted Hamiltonian are:

$$\frac{\partial H}{\partial c} = 0$$

$$-\frac{\partial H}{\partial a} = \dot{v}$$

- c) Solve the producer's problem and derive the necessary conditions for the neoclassical steady state.
- d) Draw the transition to the steady state path within a phase diagram over the space of consumption per labor and capital per labor. There is no need to give full explanation of the phase diagram, yet you must depict the direction of time-adjustments of these two variables explicitly in your diagram.
- e) Now suppose that the government decides to implement a tax on wage incomes. Thus the budget constraint of the agent becomes:

$$\dot{a} = (1-t)w + ra - c$$

where  $t$  is the tax rate on wage incomes. Re-work your solution to the optimal consumption path. How are the first order conditions towards the steady state affected?

- f) Show clearly the net effects of the tax on the new steady state equilibrium. Explain your result briefly.

5) (20 points)

The following data is from Turkish manufacturing industry

years	Average			
	Employees on wages	Employed in Production	wage costs	Value added
1980	786,995	642,633	288,871	819,884
1990	1,023,669	803,234	18,980,450	76,776,206
1997	1,138,397	896,055	1,251,982,548	6,265,312,437

- a) What is the labor value added ratio in 1980?
- b) What is the rate of exploitation in *Marxian* sense in 1980, 1990, 1997?
- c) Suppose that the value added in manufacturing were to be portrayed by a neoclassical production of the form,  $Y = K^\alpha L^{1-\alpha}$ , what would be the value of the parameter  $\alpha$ ?
- d) If you were to use the production technology depicted in (c) above under the neoclassical assumptions, would there be any exploitation? Discuss briefly.