Ec453 Theories of Growth and Development
Fall 2005, Final Exam

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1) (50 points)
Consider the neoclassical (Solow) model of exogenous growth with the following technological set up: Single output Y is produced by labor, L, and capital, K. The production technology is:

$$Y = \left[\alpha K^{-\beta} + (1-\alpha)L^{-\beta}\right]^{-\frac{1}{\beta}}$$

where K and L denote capital and labor inputs, respectively. The parameter, β, is related to the elasticity of substitution between capital and labor as $\sigma = \frac{1}{1+\beta}$.

There is no technological progress, and growth of labor is also assumed to be zero. Capital depreciates at δ. A fraction *s* of output is saved and invested for capital accumulation. For simplicity assume that stock of labor, L, is assumed to remain constant at unity.

a) Find the steady state level of capital stock, K_S*, in the Solowian long run equilibrium.
b) Now solve for the *golden rule* level of the steady state capital stock under the Solowian set-up. Show that the net rate of return to capital is zero under the golden rule. Also give an expression for the wage rate under the golden rule (you may use the known properties of the golden rule without proof).
c) Now suppose that the producers are able to have a regime change and the economy is now maximizing the total net profits (total net returns to capital). Find the new value of the capital stock under the steady sate that maximizes net profits

d) Now suppose that the consumer is acting as an intertemporal optimizer (call this the *ITO-economy*) with the standard CIES preferences:

$$U = \int_{t=0}^{\infty} e^{-\rho t} \frac{c_t^{1-\sigma} - 1}{1-\theta} dt$$

where $\rho>0$ is the subjective discount rate, and θ is the inverse of the intertemporal elasticity of substitution.

e) Using the *Hamiltonian* of this system which maximizes per capita consumption subject to the capital accumulation constraint, $\dot{K} = F(K) - \delta K - c$ find the long run equilibrium rate of growth of per capita consumption for this economy.

f) Characterize the equations for transition to steady state growth for the *ITO-economy* and show the transition to the steady state in a phase diagram. In your diagram solve for the new level of the steady state capita stock K_{ITO}* explicitly as a function of the given parameters of this model. Also in your diagram, state explicitly why certain regions of the transitional dynamics violate the so-called *Inada conditions*, and therefore are not allowed. Be sure to define the *Inada conditions*, so that I can understand what you are talking about.

g) Suppose that the elasticity of substitution between capital and labor, σ, has increased parametrically. How does this change affect the steady state value of capital? How do you interpret this result?

h) If the inverse of the intertemporal substitution elasticity, θ, is increased, how is the system of transitional dynamics affected? Are the steady state values of capital and consumption affected? Discuss.

i) Show that the consumption maximizing value of capital stock is to the right of the steady state, *i.e.*, the economy uses less capital stock and thus consumes less in comparison to the golden rule. Interpret this result and discuss its implications.

2) (10 points)
Consider the *Kaldorian* model where workers' saving rate is negative: -0.10, and investment is given from outside with I =I*. There is no government spending and the economy is closed to foreign trade.

a) Show the share of wages in output and verify that it is inversely related with the investment share of aggregate demand.
b) What is the Keynesian multiplier for this economy? How is it related to the share of profit income in aggregate income?

3) (30 points)
Consider the R&D-driven endogenous growth model with the following technology for the production of the final good:

$$Y = L_Y^{1-\alpha} \sum_i^A x_i^\alpha$$

a) Romer (1990) specifies the following relationship for the knowledge function:

$$\dot{A} = \delta L_A A$$

where A is stock of knowledge (ideas), δ is productivity of research-personnel; and L_A is the amount of labor allocated to knowledge production. Discuss the implications and shortcomings of this specification in explaining the nature and rate of growth of real world economies.

Given the shortcomings of Romer's formulation for R&D production, Jones has suggested the following:

$$\dot{A} = \delta L_A^\lambda A^\phi$$

b) Contrast the two formulations. In particular, how do you justify the Jones's treatment of R&D?
c) Find the rate of growth of the two types of the economies stated above, the *Romerian* one with the first R&D function above, and the *Jonesian*, which operates with the R&D function in section (b). Add new assumptions as you see fit. Yet, you have to assume that both economies are working with the same type of technology in the intermediates sector.
d) Find the share of labor allocated to the R&D sector in the two economies. Interpret your result.
e) It is argued that for the R&D-driven endogenous growth models, the market solution usually leads to *pareto inferior* outcomes; *i.e.*, that growth and welfare can be improved upon the market solution by choice of an appropriate tax/subsidy scheme. Comment on this argument. What are the structural reasons of this particular trait?

4) (10 points)

a) "Models of endogenous technological progress tend to assume non-diminishing returns to research effort and to the stock of domestic knowledge. As a consequence, they imply population scale effects which are counter-factual." Discuss this statement, explaining in detail the argument that is proposed. Does evidence against scale effects invalidate theories of endogenous technological progress?

b) Consider the standard neoclassical growth model, a la Solow (1956). Suppose that two countries are in a race to achieve higher rate of growth and a higher level of per capita output. Assume that both countries are identical in every respect; yet, the initial level of capital per labor in country A is twice that of country B. True or false: income per capita in country B will never reach that of country A.

c) We have seen a variety of neoclassical, Marxian and neo-Ricardian models of general equilibrium and growth. In your assessment, what are the critical distinguishing features of each paradigm? That is, what are the key characteristics that you look for when identifying each set of models from each other?

That was a fun semester, enjoy your career…