Ec453 Theories of Growth and Development
Homework: Marxian and Neoclassical Growth Contrasted

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Smallabodia is a small, private ownership economy. There is only one homogenous good, wheat, which is produced using labor and seeds of wheat alone. Land is in abundant supply, and is not considered to be a scarce good.

Currently the following data is being observed in Smallabodia: output per labor, *q*, is 10; investment per labor, *i*, is 4; and consumption per labor, *c*, is 6. The technology in use in Smalloabodia's wheat production admits that the ongoing capital-labor ratio, *k*, is 2.

1) Consider the *classical* characterization (*i,e,*, neo-Ricardian and neo-Marxian) of the Smallabodian economy where (i) workers' wages are driven to subsistence consumption; (ii) workers do not save; and (iii) capitalists do not consume. Under these conditions,
   a) State the capital output ratio, *v*; and the labor output ratio, l_q.
   b) Given that the wage rate (wages per labor) is equal to consumption per labor, find the profit rate in Smallabodia.
   c) Using information you derived thus far, find the prevailing growth rate in Smallabodia. Check the macro equilibrium using the demand-supply balance. Portray the **Marxian** equilibrium of this economy in a 4-quadrant graph of *w, r, g,* and *c*.. In particular, what is the *exploitation rate*?

2) Now we will express the macro equilibrium of Smallabodia under a *neoclassical* setting where golden rule applies, *i.e.*, per capita consumption is maximized in the long run. Suppose that the total wheat harvested is given by the (neoclassical) production function, $q=Ak^{\alpha}$, where *A* is a parameter narrating the technology of the economy. Yet there is no allowance for exogenous growth in the parameter *A*, *i.e.* $\frac{\dot{A}}{A} = 0$.

Observe that the data configuration of the economy remains the same. Thus, q=10, k= 2, r=1, and w=6. The first task is to *calibrate* this data set to the *neoclassical function* we specify above.

a) Using the above information on income distribution, *w* and *r*, find the value of the parameter α.
b) Using the data and the value of α calibrated above, state the value of the technology parameter, *A*.
c) Now, let's characterize where this economy is heading under the neoclassical assumptions. Find the *steady state neoclassical equilibrium* for Smallabodia where per capita consumption had been maximized (the golden rule applies). . In this characterization, first remember: what is the saving rate (savings out of total income), *s*? Furthermore, recall that in this model, seeds invested in the previous

year become capital stock of the current year. Otherwise capital cannot be stored and accumulated. Thus depreciation of capital, δ is 1.0 (*i.e.*, 100%).

d) What are the new values of *k*, *q*, and *v* under the **neoclassical** steady state equilibrium?

Very critical observation: observe that the capital output ration, *v*, has adjusted to a new value under the neoclassical steady state equilibrium from its initial value under the Marxian equilibrium stated above. Thus the *Harrod-Domar* s*tability condition*,

$$\frac{s}{v} = n + \delta + x$$

is satisfied via adjustments in *v*. This is the underlying distinguishing feature of neoclassical transitional growth towards the steady state long run equilibrium.

e) What are the *net* profit rate and the wage rate under **neoclassical** steady state golden rule equilibrium? Verify the unit cost price equation: $1 = w\, l_q + (1+r)\, v$

f) What is the growth rate under neoclassical steady state, *g*, what is consumption per capita, *c*? Verify the market equilibrium equation:

$$1 = c\, l_q + (1+g)\, v$$

Important observations: Observe that under the neoclassical growth rate of output per capita is driven to zero. In class we have seen that this outcome is due to *diminishing returns to capital*. This assumption on the neoclassical production function leads to *falling rate of profit*; hence, rate of capital accumulation, which otherwise is the *only* source of growth in this model, ceases and becomes nil. It has to be recalled that the hypothesis of the *falling rate of profit* is also a central component of the Marxian model, though we do not use this feature in the model described in part (a) above. In Marx, the falling rate of profit follows from cut-throat, intensive competition among capitalists, which leads to *rise of the organic composition of capital.* As capital intensity increases, labor embedded in the valuation of the product falls; therefore *the exploitation rate* also falls. Thus, in Marx the tendency for the profit rate to fall is due to falling rate of exploitation due to capitalist competition, rather than diminishing returns to capital along a production function as in the neoclassical world.

Finally, observe that in this model, Marxian growth is ensured by the fact that, $i_t = \boldsymbol{k_{t+1} > k_t}$. In the neoclassical model, on the other hand, $i = sf(k)$. (Recall that this *is* a very important distinguishing assumption of the neoclassical paradigm) Since, under the neoclassical steady state $sf(k) = \delta k$, and we also have $k_{t+1} = (1-\delta)k_t + i_t$, with $\delta = 1$, we have $i_t = \boldsymbol{k_{t+1} = k_t}$. Thus, capital accumulation has stopped under the neoclassical steady state. In contrast, the Marxian model does not invoke any characterization of a steady state…

For additional fun:

3) You may wish to sketch a procedure showing how you would derive the **Neo-Ricardian** equilibrium of this economy. Again make additional assumptions if necessary.