Ec453 Theories of Growth and Development I
Homework Set: 3

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1) In the country of Douglasia, output is produced along the production function:

$$Q = K^\alpha L^{(1-\alpha)}$$

where Q is output, and K and L are the inputs of capital and labor, respectively.

     i. Write the output per labor function for Douglasia. Find an expression of the rental rate of capital (the profit rate) and the wage rate, and the factor shares in terms of the per capita production function parameters.

     ii. Suppose that the growth rate of labor force is zero, and the rate of depreciation of capital is 2%. The Douglasians are currently saving 20% of their national output for investment purposes. Further suppose that a is a known parameter estimated to be 0.5. Calculate the steady state capital-labor ratio for Douglasia. Sketch a graph.

     iii. Calculate the per capita maximizing rate of savings for Douglasia. Find the levels of per capita consumption, savings, and the capital labor ratio under the new steady state.

     iv. Calculate the Golden Age of this system. That is find the per capita profit maximizing levels of per capita consumption, savings, and the capital labor ratio under the new steady state, and compare them with the findings in iii, that is the Golden Rule.

     v. Now suppose that a group of young graduates of Bilkent Economics has come with a non-refutable finding that in Douglasia the share of capital, i.e. a, is actually 1.00. Calculate the new steady state and draw the "transitional dynamics" of the economy under this new specification.

2) One of the classical propositions of the *Harrod-Domar* model is that, for balanced growth to exist, the following relationship must hold: *v = s/n*; where *v* is the capital output ratio, *s* is savings rate, and *n* is the population growth rate. Show that the neoclassical growth model satisfies this condition by adjustments in the capital labor ratio in its adjustments towards steady state.

3) Consider the following version of the Neoclassical growth model due to Solow (1956): Output is produced along the production function: $Q = K^{\alpha} L^{(1-\alpha)}$

where Q is output, and K and L are the inputs of capital and labor, respectively. Suppose that α is equal to 0.5. The labor force grows at the natural rate n>0, and suppose that capital doesnot depreciate.

Suppose that there is a subsistence level of consumption, c*, below which consumption will not fall,. Furthermore, saving rate is regarded not a constant but follows the following path:

   For very "low" levels of output and capital, s = s' such that k = ε (ε>0), so that a minimal amount of capital can be invested to be able to produce c*. Beyond this level of output, saving rate follows the following rule:

   s = g(k) with $\partial g / \partial k > 0$ and $\partial^2 / \partial k^2 < 0$ for c > c*;
   and
   s = s', otherwise.

Where s is saving rate per person, and y is per capita income.

Sketch out the transitional dynamics towards the steady state with the aid of a graph of output per labor and capital output ratio, and discuss the mechanisms of adjustment. Observe that you are not asked a formal analytical depiction of steady state equilibrium.