

SPO's Standard CGE Model: TURKCGE ALGEBRAIC EQUATIONS

I. Within-period system of equations

1) Price System

Composite price

$$PC_i = (1 - tva_i) \left[P_i^D \left(\frac{DC_i}{CC_i} \right) + P_i^M \left(\frac{M_i}{CC_i} \right) \right]$$

Import price

$$P_i^M = (1 + tm_i) P_i^W$$

Output price

$$PX_i = \left[P_i^D \left(\frac{DC_i}{XS_i} \right) + P_i^E \left(\frac{E_i}{XS_i} \right) \right]$$

Net (value added) price

$$PVA_i = PX_i - tn_i - \sum_j a_{ji} XS_i PC_j$$

2) Production Technology

Gross output supply

$$X_i^S = \bar{A} K^\alpha L_S^\beta L_{US}^{(1-\alpha-\beta)}$$

Intermediate Input Demand in Sector i

$$INT_i = \sum_j a_{ji} X_i^S$$

3) Labor Markets

To obtain labor demand, marginal productivity is equated with the real wage rate:

$$\left(\frac{\partial X}{\partial L} = \frac{w}{PVA}\right)$$

for skilled labor

$$\beta \cdot VA \cdot X^s = (1 + tw)\bar{w}_s L_s^D \quad \text{for } i \notin \text{public services}$$

for unskilled labor

$$(1 - \alpha - \beta)PVA = w_{us} L_{us}^D$$

Labor market clearing: Fixed nominal wages for skilled and flexible for unskilled labors

$$w_s = \bar{w}_s \quad \Rightarrow \quad UNEMP_s = \bar{L}_s^s - \sum_i L_{si}^D$$

$$\sum_i L_{us,i}^D = \bar{L}_{us}^s + UNEMP_s$$

4) Income Generation

Enterprise profits

$$RP_i = PVA_i X_i^s - (1 + tw)\bar{w}_s L_{si}^D - w_{us} L_{usi}^D$$

After-tax (net) profit income of enterprises

$$YEnet = (1 - t_k) \sum_i RP_i$$

Transfer of enterprise income to households (dividends)

$$EtrHH = YEnet - \sum_i NFI_i^G - trrow \sum_i (1 - tk) RP_i + GtrSEE + r^D \text{DomDebt}^G - r^F \text{ForDebt}^E + \text{ForBOR}^E$$

Transfer of enterprise income to public sector (public sector net factor income)

$$NFI_i^G = shrg_i RP_i$$

Profit transfers to the rest of the world

$$trrow \sum (1 - tk) RP_i$$

Private household net labor income

$$YHWnet = (1 - sstax_l) \bar{w}_S \sum_i L_{S,i}^D + w_{US} \sum_i L_{US,i}^D$$

Total Private Income

$$totYHH = YHWnet + EtrHH + GtrHH + SSITrHH + ROWtrHH$$

Net Private Income (Private Disposable Income)

$$YHnet = (1 - ty) totYHH$$

5) Public Sector Balances

Public sector aggregate revenues

$$GREV = \sum_i tn_i \cdot PX_i \cdot XS_i + \sum_i tm_i P_i^{wm} M_i + \sum_i tva_i \cdot PQ_i CC_i + ty \cdot totYHH + t_k \sum_i RP_i + \sum_i NFI_i^G$$

Public Savings

$$GSAV = GREV - GCON - r^F ForDebt^G - r^D DomDebt^G - GtrHH - GtrSEE - GtrSSI + ForBor^G$$

Government fiscal policy (primary balance objective)

$$PRIMBAL = GREV - GCON - GINV - GtrHH - GtrSEE - GtrSSI$$

$$PRIMBAL = prbrat \cdot GDP$$

Fiscal closure (choose either of the following, leaving the other free)

$$GCON = gcr \cdot GDP$$

$$GINV = gir \cdot GDP$$

*Social Security Institutions
Revenues*

$$revSSI = (tw + sstax_l) \bar{w}_S \sum_i L_{S,i}^D$$

Government transfers to SSIs

$$GtrSSI = SSITrHH - revSSI$$

6) Financial Accounts

Private savings

$$PSAV = s^P YHnet$$

Saving-Investment balance (Walras Law)

$$PSAV + GSAV + CAdef = PINV + GINV$$

7) Sectoral Demands

Private consumption by sectors

$$CD_i = cles_i \cdot \frac{PRIVCON}{PC_i}$$

where $PRIVCON = (1 - s^P) YHnet$

Private investment demand (by sector of origin)

$$IDP_i = iples_i \cdot \frac{PINV}{PC_i}$$

Government consumption by sectors

$$GD_i = gles_i \cdot \frac{GOVCON}{PC_i} \quad i \notin \text{public services}$$

$$GD_{PUBSERV} = \bar{w}_S \cdot L_{S, PUBSERV}^D$$

Government investment demand (by sector of origin)

$$IDG_i = igles_i \cdot \frac{GINV}{PC_i}$$

8) Rest of the World

$$CA_{def} = \sum P_i^W E_i + ROWtrHH + ForBor^E + ForBor^G \\ - \left[\sum P_i^W M_i + trrow \sum (1 - tk) RP_i + r^F ForDebt^E + r^F ForDebt^G \right]$$

Determination of export supply

$$X_i^s = CET(DC_i, E_i)$$

with export-domestic good ratio:

$$\frac{E_i}{DC_i} = \left(\frac{P_i^W}{PD_i} \right)^\gamma \left(\frac{\chi}{1 - \chi} \right)^\gamma$$

Determination of import demand

$$CC_i = CES(DC_i, M_i)$$

with import-domestic good ratio:

$$\frac{M_i}{DC_i} = \left(\frac{(1 + tm_i) P_i^W}{PD_i} \right)^\epsilon \left(\frac{\delta}{1 - \delta} \right)^\epsilon$$

9) Market Clearing

$$CC_i = CD_i + GCON_i + IDP_i + IDG_i + INT_i$$

II. Dynamics

1) Evolution of government and private debt

Government budget deficit (Public Sector Borrowing Requirement-PSBR)

$$PSBR = GREV - GCON - GINV - r^F ForDebt^G - r^D DomDebt^G - GtrHH - GtrSEE - GtrSSI$$

Government's Foreign Borrowing is a ratio of PSBR:

$$ForBor^G = (gborrat)PSBR; \text{ thus } DomBor = (1 - gborrat)PSBR$$

Government Domestic Debt:

$$DomDebt_{t+1} = DomDebt_t + DomBor_t$$

Government Foreign Debt

$$ForDebt^G_{t+1} = ForDebt^G_t + ForBor^G_t$$

Private foreign debt

$$ForDebt^P_{t+1} = ForDebt^P_t + ForBor^E_t$$

2) Rules to update exogenous variables and policy variables:

Increase sectoral capital stocks by investments (by sector of destination) net of depreciation

Increase labor supplies by population growth

Increase technology indexes by TFP rates

The following variables are updated as a fixed ratio to the expenditures of respective units:

$ForBor^E$ and $ForBor^G$, $GtrHH$, $GtrSEE$

The following are updated as a ratio to the GDP:

$GtrSSI$, $ROWtrHH$, $SSltrHH$