Chapter 1

Public Goods

1.1 How Many Streetlights?

Suppose that 10 people live on a street and each of them is willing to pay \$2 for each extra streetlight, regardless of the number of streetlights provided. If the cost of providing x streetlights is given by $c(x) = x^2$, what is the Pareto efficient number of streetlights to provide?

1.2 Thelma and Louise

Thelma and Louise are neighbours. During the winter it is impossible for a snowplow to clear the street in front of Thelma's house without clearing the front of Louise's. Thelma's marginal benefit from snowplowing services is $12 - z$ where z is the number of times the street is plowed. Louise's marginal benefit is $8 - 2z$. The marginal cost of getting the street plowed is \$16 . Sketch the two marginal benefit schedules and the aggregate marginal benefit schedule. Draw in the marginal cost schedule, and find the efficient level of provision for snowplowing services?

1.3 Tunus-Merkez

Shuttle buses from Bilkent University Campus to the centrum of Ankara is considered as a public service which is provided by the university. There are 1000 students using the buses, each having a personal total benefit function $TB = 0.001B^2 + 0.1B$, where B is the number of buses per day. [1] University's cost function is $TC = 2B^2$.

- a. Explain why shuttle buses are characterized as a public service above? Give some justifications.
- b. Assuming that the shuttle service is provided privately, calculate how many bus trips will be provided.
- c. Now calculate the publicly provided amount of the same service.
- d. Compare the benefits to the society in parts (b) and (c), report any differences you have observed.

1.4 Personalized Prices

Assume that the society consists of three individuals A, B and C. Their demand functions for the number of streetlights (a public good) are represented by

$$Q_A = 10 - 2P$$
$$Q_B = 15 - 3P$$
$$Q_C = 24 - 6P$$

- a. if the marginal cost of providing streetlights is constant at \$8 , how many streetlights should be provided so that the provision of this public good is Pareto efficient?
- b. How much each individual is willing to pay at the equilibrium level of street lights?

[1] You may realize that marginal benefit function is upward-sloping in this example. Then, from an economic point of view, this example is degenerate.

- c. What are the total tax payments made by each individual?

1.5 Homeland Security

Table 1.1 provides data on the marginal benefits of three consumers who desire security protection in a community. In the table, the marginal benefits for up to 4 security guards per week are shown for each of the consumers A, B and C.

Table 1.1: Benefit Matrix

	Number of guards			
	1	2	3	4
MB_A	300	250	200	150
MB_B	250	200	150	100
MB_C	200	150	100	50

Suppose that the weekly cost per security guard is $450.

- a. How many security guards will be hired and how much each individual pay for the security service?
- b. Suppose the price of hiring a security guard increases from $450 to $600 . Show how this will affect the number of security guards hired and the price paid by each consumer

1.6 Woody and Mia

Woody and Mia consider buying a TV receiver for their living room[2]. Their discounted individual benefits from watching TV are equivalent to $150 each. The TV set costs $200. Examine the given case as a simple game.

[2]Remember that the livingroom is the only common place of a “classical” household

Chapter 2

Externalities

2.1 A Consumption Externality -1- Exchange Economy

There are two individuals in an economy whose utility functions are given as $U = U(x_1, y_1)$ and $V = V(x_2, y_2, y_1)$. The two individuals consume whole endowments of x and y in that economy. Solve for the Pareto Optimal allocations and examine the externalities within this setup, given the blow characterization of individuals' preferences.

$$U = U(x_1, y_1) \tag{2.1}$$

$$\frac{\partial U}{\partial x_1} > 0 \tag{2.2}$$

$$\frac{\partial U}{\partial y_1} > 0 \tag{2.3}$$

$$V = V(x_2, y_2, y_1) \tag{2.4}$$

$$\frac{\partial V}{\partial x_2} > 0 \tag{2.5}$$

$$\frac{\partial V}{\partial y_2} > 0 \tag{2.6}$$

$$\frac{\partial V}{\partial y_1} > 0 \tag{2.7}$$

Equations (2.2), (2.3), (2.5) and (2.6) correspond to positive marginal utilities, which is a well-known construct in microeconomics. Equation (2.7), on the other hand, tells us that the consumption of good y by individual U affects individual V in a positive manner.This indicates a positive consumption externality.[1]

The feasibility conditions are written as:

$$x_1 + x_2 = x \tag{2.8}$$

and

$$y_1 + y_2 = y \tag{2.9}$$

In order to derive the necessary conditions for Pareto optimality, one should cast the following program:
Maximize

$$V(x_2, y_2, y_1) \tag{2.10}$$

subject to:

$$U(x_1, y_1) = \bar{U} \tag{2.11}$$

$$x_1 + x_2 = x \tag{2.12}$$

$$y_1 + y_2 = y \tag{2.13}$$

The Lagrangean function is constructed as:

$$L = V(x_2, y_2, y_1) + \lambda\left[\bar{U} - U(x_1, y_1)\right] + \mu_1\left[x - x_1 - x_2\right] + \mu_2\left[y - y_1 - y_2\right] \tag{2.14}$$

[1]Consider V as a mother and U as her kid. When the kid enjoys more food (i.e. good y) the mother would derive a higher level of utility.

The First Order Conditions are then computed:

$$\frac{\partial L}{\partial x_2} = \frac{\partial V}{\partial x_2} - \mu_1 = 0 \tag{2.15}$$

$$\frac{\partial L}{\partial y_2} = \frac{\partial V}{\partial y_2} - \mu_2 = 0 \tag{2.16}$$

$$\frac{\partial L}{\partial x_1} = -\lambda \frac{\partial U}{\partial x_1} - \mu_1 = 0 \tag{2.17}$$

$$\frac{\partial L}{\partial y_1} = \frac{\partial V}{\partial y_1} - \lambda \frac{\partial U}{\partial y_1} - \mu_2 = 0 \tag{2.18}$$

Using Equations (2.15) and (2.16), we obtain:

$$\frac{\frac{\partial V}{\partial x_2}}{\frac{\partial V}{\partial y_2}} = MRS_2^{xy} = \frac{\mu_1}{\mu_2} \tag{2.19}$$

Similarly, using Equations (2.17) and (2.18):

$$\frac{-\lambda \frac{\partial U}{\partial x_1}}{\frac{\partial V}{\partial y_1} - \lambda \frac{\partial U}{\partial y_1}} = \frac{\mu_1}{\mu_2} \tag{2.20}$$

is obtained. However, this expression in not intuitive enough. We should do some math.

Equation (2.22) is the same as Equation (2.20) written as its twice-reciprocal:

$$\frac{\mu_1}{\mu_2} = \frac{-\lambda \frac{\partial U}{\partial x_1}}{\frac{\partial V}{\partial y_1} - \lambda \frac{\partial U}{\partial y_1}} \tag{2.21}$$

$$= \frac{1}{\frac{\frac{\partial V}{\partial y_1} - \lambda \frac{\partial U}{\partial y_1}}{-\lambda \frac{\partial U}{\partial x_1}}} \tag{2.22}$$

$$= \frac{1}{\frac{\lambda \frac{\partial U}{\partial y_1} - \frac{\partial V}{\partial y_1}}{\lambda \frac{\partial U}{\partial x_1}}} \tag{2.23}$$

$$= \frac{1}{\frac{\frac{\partial U}{\partial y_1}}{\frac{\partial U}{\partial x_1}} - \frac{1}{\lambda}\frac{\frac{\partial V}{\partial y_1}}{\frac{\partial U}{\partial x_1}}} \tag{2.24}$$

From Equation (2.15) we know that:

$$\mu_1 = \frac{\partial V}{\partial x_2} \tag{2.25}$$

and Equation (2.17) tells that:

$$-\lambda \frac{\partial U}{\partial x_1} = \mu_1 \tag{2.26}$$

Rearranging Equation (2.26):

$$\lambda = -\frac{\mu_1}{\frac{\partial U}{\partial x_1}} \tag{2.27}$$

Using Equations (2.25) and (2.27):

$$\lambda = -\frac{\frac{\partial V}{\partial x_2}}{\frac{\partial U}{\partial x_1}} \tag{2.28}$$

Now, substitute Equation (2.28) into (2.24):

$$\frac{\mu_1}{\mu_2} = \frac{1}{\frac{\frac{\partial U}{\partial y_1}}{\frac{\partial U}{\partial x_1}} + \frac{\frac{\partial U}{\partial x_1}}{\frac{\partial V}{\partial x_2}}\frac{\frac{\partial V}{\partial y_1}}{\frac{\partial U}{\partial x_1}}} \tag{2.29}$$

Perform the necessary cancellations:

$$= \frac{1}{\frac{\frac{\partial U}{\partial y_1}}{\frac{\partial U}{\partial x_1}} + \frac{\frac{\partial V}{\partial y_1}}{\frac{\partial V}{\partial x_2}}} \tag{2.30}$$

Rewriting Equation (2.20):

$$\frac{\frac{\partial V}{\partial y_2}}{\frac{\partial V}{\partial x_2}} = MRS_2^{yx} = \frac{\mu_2}{\mu_1} \tag{2.31}$$

We know that:

$$\frac{\frac{\partial U}{\partial y_1}}{\frac{\partial U}{\partial x_1}} = MRS_1^{yx} \tag{2.32}$$

The reciprocal of Equation (2.30) gives us:

$$\frac{\mu_2}{\mu_1} = \frac{\frac{\partial U}{\partial y_1}}{\frac{\partial U}{\partial x_1}} + \frac{\frac{\partial V}{\partial y_1}}{\frac{\partial V}{\partial x_2}} \tag{2.33}$$

Finally, we obtain:

$$MRS_2^{yx} = MRS_1^{yx} + \frac{\frac{\partial V}{\partial y_1}}{\frac{\partial V}{\partial x_2}} \tag{2.34}$$

as the condition for Pareto Optimality. Remember that the condition for Pareto optimality is:

$$MRS_2^{yx} = MRS_1^{yx} \tag{2.35}$$

in the absence of externalities.

The difference between the two conditions implies that the consumption of y by individual U will increase if the externality is positive and decrease if the externality is negative (You can verify this).[2]

2.2 Around Bilkent

Bilkent University Campus, Real Shopping Centre, Sports International, and Tepe Knauf are located near each other and the cost function of each economic entity is given as follows:

[2] Imagine the attitudes of a child-caring mother.

Real Shopping Centre:

$$C(m) = Am^2 - Bl \tag{2.36}$$

Sports International:

$$C(s) = Ds^2 + Em + F \cdot k \tag{2.37}$$

Bilkent University Campus:

$$C(l) = Gl^2 - Hm + Jk - Ks \tag{2.38}$$

Tepe Knauf:

$$C(k) = Nk^2 \tag{2.39}$$

(Assume A, B, D, E, F, G, H, J, K, and N are positive.)

Where:

m:a variable representing the quantity of all goods sold in the shopping centre
s:amount of sports and recreational activities provided
l:index of life quality in the university campus
k:amount of Tepe-Knauf products
Prices of these goods and services are P_m, P_s, P_l, P_k respectively and determined under competitive conditions.

Now you are supposed to answer the following based on the given information:

- a. Define what an externality is. For the given setting indicate the externalities. Are they negative or positive in their nature? Give possible reasons for their negativity or positivity.

- b. Under perfectly competitive conditions how much each economic entity provides to its customers/students/members etc.? Is this solution Pareto efficient? Explain.

- c. If your answer to (b) is “no” then what would the Pareto efficient levels of provision be?

- d. Are the levels you found in parts (b) and (c) different. Why? Explain in a short paragraph (Do not argue mathematically).

2.3 Warming Up

Demand curve for widgets is given as $x = 10 - p$, where p and x are the price and quantity of widgets, respectively. Marginal private cost of producing widgets is \$5. Production of widgets implies a marginal damage of 2 dollars per unit. Compute the optimal amount of widget production in the absence of government intervention. Next, find the socially optimal production scheme. What is the net gain of society by moving from the market solution to the socially optimal one? Sketch your findings on a simple graph.

2.4 Bees and Apples

Suppose that an apple orchard is located next to a beekeeper. When the orchard produces a apples, and the beekeeper produces h units of honey, the cost function for the apple orchard will be $C_a(a) = a^2$ and the cost function for the beekeeper will be $C_h(h) = h^2 - a$. Suppose that the price of apples is 4 dollars per unit and that the price of honey is 8 dollars per unit.

- a. Supposing that each firm acts independently, find the amount of apples and honey that will be produced.

- b. Are the production levels that you have found in part (a) Pareto efficient levels? If not, find the Pareto efficeint level of output for the apple orchard and the bee farm.

- c. Suppose government would like to use taxes or subsidies to correct for the externality. Find the optimal level of taxes or the subsidies.

2.5 Bees, Apples and Recreation

Suppose that an apple orchard, a beekeper and a recreation club are located next to each other. Cost functions of these three businesses are given as $C_A = \frac{A^2}{100} + H$, $C_H = \frac{H^2}{100} + A$ and $C_S = \frac{S^2}{50} - 2H$, respectively; where A, H and S are the amounts of apples, honey and recreational services provided. You may notice that there exist a number of externalities in this setup. Given that the prices of the commodities are $P_A = 3$, $P_H = 5$ and $P_S = 6$ elaborate the following questions:

- a. Figure out the market equilibrium. Is this solution socially optimal?
- b. If the three businesses consider the inherent externalities in their decision making process, in what direction the provided quantities are expected to change?
- c. Figure out the merger solution for these businesses.
- d. Propose a Pigouvian tax/subsidy scheme which ensures the solution you found in part (d). What is the net change in government's budget position having this scheme implemented?

2.6 University-Industry Cooperation

A university produces research in the areas of natural sciences and engineering. Its cost function is summarized as $C_u(n) = n^2$ where n is the volume of scientific research. At the same time, a firm producing electrical appliances has a cost function, which is expressed as $C_f(x, n) = x^2 - xn$, where x is the production of appliances. Price of each unit of research is \$1 and price of each appliance is \$2. Given this information, first discuss the nature of interaction between the university and firm. Then,

- a. Find the market solution for the two institutions.
- b. Find the socially optimal solution. Is this solution identical to the market solution?
- c. Design a subsidy scheme to provide adequate incentives in this setup which ensures the socially optimal equilibrium.